[redacted]

11p.

NASA TT F-8218

N65-22581
(ACCESSION NUMBER)

11
(PAGES)

(NASA CR OR TMX OR AD NUMBER)

(THRU)

1
(CODE)

25
(CATEGORY)

FACILITY FORM 602

ON ONE-DIMENSIONAL AUTOMODEL WAVES MOVING IN

A PLASMA ALONG A MAGNETIC FIELD

by B. A. Tverskoy

GPO PRICE $

OTS PRICE(S) $

Hard copy (HC) $1.00

Microfiche (MF) .50

[redacted]

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON May 1962

[redacted]

[redacted]

DRAFT TRANSLATION

NASA TT F-8218

MAY 1 4 1962

ON ONE-DIMENSIONAL AUTOMODEL WAVES MOVING IN A PLASMA ALONG A MAGNETIC FIELD

(Ob odnomernykh avtomodel'nykh volnakh rasprostranyayushchikhsya v plazme vdol' magnitnogo polya)

Zhurnal Eksper. i teoreticheskoy fiziki (ZhETF)
Tom 42, vyp. 3, pp. 833-838,
A. N. SSSR, Moskva, 1962.

by B. A. Tverskoy

ABSTRACT

22581 1043

A one-dimensional steady state pulse moving along a magnetic field in a cold plasma is considered. The calculations are performed in the nonrelativistic single-particle approximation without taking into account the collisions, the plasma being assumed quasi-neutral. An exact solution of the corresponding equation is presented. The shape of the pulse and the distribution of velocity are investigated The lines of force of the magnetic field and the particles' trajectories in the pulse are found to be spirals. Determined also is the dependence of the Mach number on the wave energy. It is shown that even small oscillations of the magnetic field cause an appreciable acceleration of the electron component of the pulse at its maximum. Most of the wave energy is in this case concentrated in the kinetic energy of electrons.

COVER-TO-COVER TRANSLATION

The one-dimensional steady state flows of rarefied plasma moving in the direction perpendicular to the magnetic field have been studied in detail by a series of authors [1 – 2]. In the present work we shall consider steady waves, travelling along the field.

Let us postulate that because of a certain initial plasma perturbation, there occurs a plane pulse propagating along the field.

The effect of initial conditions weakens in time, and the shape of the pulse is determined by the non-linear effects — dispersion and dissipation. The latter may be linked (aside from Coulomb scattering) with the averaging of the electromagnetic particle acceleration in the wave by the unperturbed thermal velocities, and also with the pulse's instability.

Because of the complexity of the full examination of such a problem, the investigation is usually broken into two stages : first the steady-state pulse shape is found without accounting dissipation, then the dissipative effects are studied for a given shape of the wave.

It may result that in the absence of dissipation the shape of the pulse does not vary at great distances from the source and depends for an assigned unperturbed state of the plasma on the pulse rate, which in its turn is unilaterally linked with the wave energy $\mathcal{E}$.

Let us find the pulse shape as function of $\mathcal{E}$ under the following assumptions : a) the plasma is sufficiently rarefied for the Coulomb scattering to be neglected ; b) the thermal velocities of the unperturbed particles are small in comparison with the wave velocity, and therefore one may take advantage of a single-particle approximation ; c) the particle velocities in the wave are small in comparison with the speed of light.

1. BASIC EQUATIONS

In the indicated formulation the problem is described by a system of electron and ion motion equations, continuity equations and Maxwellian equations for self-consistent fields. Let us introduce the following designations : **V** and **v**, **N** and **n** are respectively the velocities and densities of ions and electrons, **E** and **H** are the electric and the magnetic field, **m** is the mass of ion, μ is the electron to ion mass ratio, e is the absolute magnitude of the charge of electron, c is the speed of light. For the sake of simplicity the ions shall be considered uninegative.

The problem's equations are as follows :

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\,\mathbf{V} = \frac{e}{m}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{VH}]\right\},$$

$$-\mu\left\{\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{v}\right\} = \frac{e}{m}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right\}, \tag{1}$$

$$\partial N/\partial t + \operatorname{div} N\mathbf{V} = 0, \quad \partial n/\partial t + \operatorname{div} n\mathbf{v} = 0, \tag{2}$$

$$\operatorname{rot}\mathbf{H} = 4\pi e c^{-1}(N\mathbf{V} - n\mathbf{v}), \tag{3}$$

$$\operatorname{rot}\mathbf{E} = -c^{-1}\,\partial\mathbf{H}/\partial t, \tag{4}$$

$$\operatorname{div}\mathbf{H} = 0, \tag{5}$$

$$\operatorname{div}\mathbf{E} = 4\pi e(N - n). \tag{6}$$

Let us direct an axis x along the undisturbed field H_0, which is presumed uniform. We shall seek the solution in a form of a stabilized plane wave travelling in the positive direction of the axis x with a velocity U. At the same time, all magnitudes depend only on a single variable $\xi = x - Ut$, so that $\partial/\partial y = \partial/\partial z = 0$, $\partial/\partial x = d/d\xi$, At $\xi = -\infty$ (unperturbed plasma) $\mathbf{E}$, H_y, H_z, $\mathbf{V}$ and $\mathbf{v}$ equal zero, $H_x = H_0$, $n = N = N_0$.

The equations (2), (4) and (5) are directly integrated :

$$N = N_0 U/(U - V_x), \quad n = N_0 U/(U - v_x), \tag{7}$$

$$E_y = UH_z/c, \quad E_z = -UH_y/c, \tag{8}$$

$$H_x = H_0 = \text{const.} \tag{9}$$

Substituting (7) – (9) into (1), we are assured that the y- and z– components of the current $c(N\mathbf{V} - n\mathbf{v})$ are total differentials. Hence, and from (3) we find that

$$H_z = -\frac{H_0}{U}M^2(V_z + \mu v_z), \quad H_y = -\frac{H_0}{U}M^2(V_y + \mu v_y), \tag{10}$$

where

$$M = \sqrt{4\pi N_0 m}\,U/H_0 \tag{11}$$

is the magnetic Mach number.

Let us introduce the dimensionless variables

$$s = \frac{eH_0}{mcU}\xi, \quad \mathbf{h} = \frac{\mathbf{H}}{H_0}, \quad \varepsilon = \frac{c}{U}\frac{E_x}{H_0}, \quad \mathbf{W} = \frac{\mathbf{V}}{U}, \quad \mathbf{w} = \frac{\mathbf{v}}{U}$$

and let us postulate

$$1 - W_x = \Theta, \quad 1 - w_x = \vartheta.$$

Upon substitution of the obtained integrals the equations are reduced to the system

$$\Theta\frac{d\Theta}{ds}=\varepsilon-M^2\mu\,(W_y w_z-W_z w_y),\quad -\mu\Theta\frac{d\vartheta}{ds}=\varepsilon+M^2\,(W_y w_z-W_z w_y),\tag{12}$$

$$\frac{dW_y}{ds}=-\left(\frac{1}{\Theta}-M^2\right)W_z+\mu M^2 w_z,\quad \mu\frac{dw_y}{ds}=\left(\frac{1}{\vartheta}-\mu M^2\right)w_z-M^2W_z,\tag{13}$$

$$\frac{dW_z}{ds}=\left(\frac{1}{\Theta}-M^2\right)W_y-\mu M^2 w_y,\quad \frac{dw_z}{ds}=-\left(\frac{1}{\vartheta}-\mu M^2\right)w_y+M^2W_y,\tag{14}$$

$$\beta^2\,d\varepsilon/ds=\Theta^{-1}-\vartheta^{-1}\quad (\beta^2=H_0^2/4\pi MN_0c).\tag{15}$$

Let us seek the solution for which Θ and ϑ are positive everywhere (particles are not reflected from the wave). In the nonrelativistic case ($\beta^2\ll 1$) we may deduce from (15) that the quasineutrality condition $\Theta-\vartheta\ll\Theta$[1]. is fulfilled (Inasmuch as **the small parameter β^2 figures in (14)** at the senior derivative, the **indicated conclusion requires a subsequent verification, which** will be effected at the end of part 2). Let us assume $\Theta-\vartheta=\beta^2\psi$ and let us expand (12) – (15) in powers β^2. In the zero approximation we must assume $\Theta=\vartheta$. Substracting equations (12) from one another, we exclude ε. The problem is thus reduced to the system of five equations

$$\begin{gathered}\Theta\frac{d\Theta}{ds}=-M^2\,(W_y w_z-W_z w_y),\\ \frac{dW_y}{ds}=-\left(\frac{1}{\Theta}-M^2\right)W_z+\mu M^2 w_z,\quad \mu\frac{dw_y}{ds}=\left(\frac{1}{\Theta}-\mu M^2\right)w_z-M^2W_z,\\ \frac{dW_z}{ds}=\left(\frac{1}{\Theta}-M^2\right)W_y-\mu M^2 w_y,\quad \mu\frac{dw_z}{ds}=-\left(\frac{1}{\Theta}-\mu M^2\right)w_y+M^2W_y.\end{gathered}\tag{16}$$

The electric field ε is determined as

$$\varepsilon=(1-\mu)\,\Theta^2d^2\Theta^2/ds^2,$$

and the correction is

$$\psi=-\tfrac{1}{2}\Theta^2\,d^2\Theta^2/ds^2.\tag{17}$$

Introducing the complex variables

$$P=W_y+iW_z,\qquad Q=w_y+iw_z,\tag{18}$$

we may write (16) in a more complex form :

..../..

$$\begin{aligned} \Theta d\Theta/ds &= \operatorname{Im} PQ^*, \\ dP/ds &= (\Theta^{-1} - M^2)\, iP - \mu M^2 iQ, \\ \mu dQ/ds &= -(\Theta^{-1} - \mu M^2)\, iQ + M^2 iP. \end{aligned} \tag{19}$$

At $s \to +\infty$ we shall have $\Theta = 1$, $P = Q = 0.$

The integration of the system (19) may be reduced to the quadrature with the aid of the substitution

$$P = p \exp\left\{i \int K(s)\, ds\right\}, \quad Q = (q + iq_1) \exp\left\{i \int K(s)\, ds\right\}, \tag{20}$$

where p, q, q_1, K are real functions.

Substituting (20) into (19) and separating the real and the imaginary parts, we shall obtain :

$$\Theta d\Theta/ds = -q_1 p, \tag{21}$$

$$dp/ds = \mu M^2 q_1, \tag{22}$$

$$[illegible] = \mu M^2 q/p, \tag{23}$$

$$\mu dq/ds = (\Theta^{-1} - \mu M^2)\, q_1 \tag{24}$$

$$\mu dq_1/ds + \mu K q = -(\Theta^{-1} - \mu M^2)\, q + M^2 p. \tag{25}$$

Assuming $\sqrt{\mu} M = \alpha$, $p = \alpha\varphi$, we shall obtain from (21) and (22) :

$$\Theta = \sqrt{1 - \varphi^2}, \tag{26}$$

and from (22) – (24) and (26) :

$$q = (\arcsin\varphi - \alpha^2\varphi)/\mu\alpha, \tag{27}$$

$$K = \frac{1}{\sqrt{1-\varphi^2}} - \frac{1}{\mu}\,\frac{\arcsin\varphi}{\varphi}. \tag{28}$$

Substituting these results into (25), we shall determine $q_1(\varphi)$:

$$q_1(\varphi) = \pm \frac{1}{\mu\alpha}\left[2\int_0^{\varphi} \frac{\arcsin^2\varphi}{\varphi}\, d\varphi - (1+\mu)\arcsin^2\varphi + \right.$$

$$\left. + 2\alpha^2 (1+\mu)\left(1 - \sqrt{1-\varphi^2}\right)\right]^{1/2}. \tag{29}$$

The spatial dependence $\varphi(s)$ is determined by quadrature from (29) and (22).

2. WAVES OF AVERAGE AND LOW INTENSITY

Let us examine the waves in which $\alpha^2 \ll 1$ (т. е. $M^2 \ll 1/\mu$). It appears that at the same time $0 \leqslant \varphi^2 \leqslant \varphi_0^2 \ll 1$, so that arc sin φ and $\sqrt{1-\varphi^2}$ may be exp ded in series with a precision to terms of the second order.

Limiting ourselves in the coefficient at φ^4 by the term of the zero order by α and μ, we shall obtain

$$q_1(\varphi) = \pm\varphi\mu^{-1}\Lambda(M)\sqrt{1-\varphi^2/6\alpha^2\Lambda^2(M)}, \tag{30}$$

where

$$\Lambda(M) = \sqrt{(M^2-1)/M^2+\mu}. \tag{31}$$

For the determination of the spatial dependence φ, we shall select the coordinates' origin in such a way that at $s = 0$. Then, we shall have from (22) and (30):

$$\varphi = \sqrt{6}\,\alpha\Lambda \Big/ \operatorname{ch}\frac{M\Lambda s}{\sqrt{\mu}}. \tag{32}$$

Limiting ourselves by the terms of the lowest orders by α and μ, we find

$$\begin{aligned} p &= \sqrt{6}M^2\Lambda(M)\mu \Big/ \operatorname{ch}\frac{M\Lambda}{\sqrt{\mu}}s, \\ q &= \frac{\sqrt{6}\Lambda}{\mu} \Big/ \operatorname{ch}\frac{M\Lambda}{\sqrt{\mu}}s, \\ q_1 &= \sqrt{\frac{6}{\mu}}\,M\Lambda^2 \operatorname{sh}\frac{M\Lambda}{\sqrt{\mu}}s \Big/ \operatorname{ch}^2\frac{M\Lambda}{\sqrt{\mu}}s, \\ K &= -1/\mu, \\ W_x &= 3\mu^2 M^4\Lambda^4/\operatorname{ch}^2\frac{M\Lambda}{\sqrt{\mu}}s. \end{aligned} \tag{33}$$

The precision estimate indicates, that for $M \leqslant 10$, the error does not exceed 3 percent.

It may be seen from (33) and (17) that the quasineutrality condition is valid at the fulfillment of the inequality

$$6\mu\beta^2M^6 \ll 1. \tag{34}$$

The nonrelativism condition for electrons requires that

$$\beta M\Lambda \ll \mu. \tag{35}$$

The comparison of (34) and (35) leads to the conclusion that the requirement $\mu M^2 \ll 1$ is an automatic consequence of nonrelativism and quasineutrality.

Returning to variables **V**, **v**, **H**, let us write their dependence on ξ :

$$V_x = v_x = 3\mu^2 M^5 \Lambda^4 U_0 \Big/ \mathrm{ch}^2 \frac{\Lambda}{\sqrt{\mu}} \frac{\xi}{\xi_0} . \tag{36}$$

$$V_y + iV_z = \sqrt{6} M^3 \Lambda \mu U_0 \exp\left\{- i \frac{\xi}{M\mu\xi_0}\right\} \Big/ \mathrm{ch} \frac{\Lambda}{\sqrt{\mu}} \frac{\xi}{\xi_0} , \tag{37}$$

$$v_y + iv_z = \sqrt{6}\, \Lambda M U_0 \times$$

$$\times \exp\left\{- i \frac{\xi}{M\mu\xi_0} + i \sqrt{\mu}\, M \Lambda\, \mathrm{th} \frac{\Lambda}{\sqrt{\mu}} \frac{\xi}{\xi_0}\right\} \Big/ \mu\, \mathrm{ch} \frac{\Lambda}{\sqrt{\mu}} \frac{\xi}{\xi_0} , \tag{38}$$

$$H_y + iH_z = - M^2 H_0 \sqrt{6}\, \Lambda \exp\left\{- i \frac{\xi}{M\mu\xi_0} + i \sqrt{\mu}\, M \Lambda\, \mathrm{th} \frac{\Lambda}{\sqrt{\mu}} \frac{\xi}{\xi_0}\right\} , \tag{39}$$

where $\xi_0 = mcU_0/eH_0$ and $U_0 = H_0/\sqrt{4\pi N_0 m}$ depend on M.

The pulse width

$$\delta = \frac{\sqrt{\mu}\, mcU_0}{eH_0} \frac{M}{[M^2(1+\mu) - 1]^{1/2}} \tag{40}$$

approaches at great M a constant boundary — **the mean geometric** among the **electron and ion Larmor radii** with velocities U_0 in the field H_0. In case of weak waves the pulse tends to spread : at M = 1 its width equals that to the ion Larmor radius, while at $M^2 - 1/(1 + \mu)$ the width $\delta \rightarrow \infty$.

It may be seen from (36) through (39) that the magnetic field and the particle trajectory inside the pulse curve into a spiral with a spacing $M\mu\xi_0$. With the wave intensity increase the spiral's spacing also increases. It is minimum for low wave intensities. Let us also note, as may be seen from the results obtained, that the solutions in linear approximation, when **V** and **H** lie in a single plane (Alfvén waves), may only exist during a limited time interval. The nonlinear effects lead to the twisting of the lines of force and of particle trajectories.

It follows from (36) that a certain concentration in the particle density takes place in the pulse maximum. Formula (38) shows that in the leading edge of the pulse electrons are characterized by a phase lag in respect to ions, while in the trailing edge, to the contrary, they are in advance of them.

As it nears the pulse maximum the particle accelerates in a plane perpendicular (transverse) to H_o. The maximum transverse ion velocity

$$V_0 = \sqrt{6}\, M^3 \Lambda \mu U_0$$

does not reach the magnitude U_o at $\mu M^2 \ll 1$. At the same time, the maximum velocity of electrons

$$v_0 = \sqrt{6}\, \Lambda M U_0 / \mu \qquad (41)$$

exceeds U_o by $\mu^{-1/2}$ times even at comparatively small M (M = 1). Conseuqnetly, the examined type of waves, contrary to transverse waves [1], mostly accelerates electrons. At $M \to 1 - \mu$, the ion and electron acceleration approaches zero thanks to the factor Λ.

The physical cause of acceleration amounts to the fact that separate ions, flying into the examined field with a velocity U, freely pass through the wave, while electrons ought to have been reflected from it. However, by the strength of quasineutrality the ions pull the electrons through the potential barrier, imparting them the necessary kinetic energy. As may be seen from (38) and (39), the electrons move then along the lines of force of the perturbed field. Inasmuch as this is basically created at the expense of electron motion, the field is force-free ($\mathbf{H} \parallel \mathrm{rot}\, \mathbf{H}$).

3. THE MACH NUMBER AS A FUNCTION OF WAVE ENERGY

The preceding results give the field H and the velocities **V** and **v** as functions of the Mach number M. In case of longitudinal waves, when the direction of wave propagation coincides with that of motion, the Mach number has a clear physical sense, and it is linked with the source to small perturbation velocity ratio. In case of transverse waves (to which the above-examined pulse is related), there is no such simple correspondence between the Mach number and the velocity of the source. Thus, its energy constitutes a better physical characteristic of the wave.

In case of a steady motion, an unambiguous correspondence takes place between M and the energy $\mathcal{E}$ over 1 cm^2 of the wave front. It may be seen from (37) and (38) that the kinetic energy is basically concentrated in the electron component, so that

$$d\mathscr{E}_{\kappa} = \frac{3\Lambda^2 M^2 U_0^2 m N_0}{\mu\, \mathrm{ch}^2 (\Lambda\xi/\sqrt{\mu}\xi_0)}\, d\xi. \qquad (42)$$

The electric energy $\mathcal{E}_E$ appears to be low in comparison with the magnetic energy $\mathcal{E}_H$, and $\mathscr{E}_H$, a $d\mathscr{E}_H = \mu M^2 d\mathscr{E}_{\kappa} \ll d\mathscr{E}_{\kappa}$.

Thus, the fundamental part of the energy of the nonrelativistic quasi-neutral pulse is concentrated in the kinetic energy of electrons.

Integrating, we find

$$6\Lambda M^2 \mathscr{E}_0/\sqrt{\mu}, \qquad (43)$$

where $\mathscr{E}_0 = H_0^2 mcU_0/8\pi eH_0$ **is the magnetic energy** of the unperturbed field in a column ξ_0 long, and of 1 cm^2 cross section.

Hence, taking into account the determination Λ (31), we shall find

$$M^2 = 1 + \sqrt{1 + \frac{\mu}{36}\left(\frac{\mathscr{E}}{\mathscr{E}_0}\right)^2}/2(1+\mu). \qquad (44)$$

Our formulae are valid at $\mu M^2 \ll 1$ and, consequently, at $\mathscr{E} \ll 6\mathscr{E}_0/\mu^{1/2}$. The Mach number depends little on $\mathcal{E}$: thus, at $\mathcal{E} \to 0$ the number $M = 1/(1+\mu)$, the value $M = 1$ is already reached at $\mathcal{E} = 12\,\mathcal{E}_0$. It may be considered within broad limits, that $M = 1$, and $\Lambda(M) = \sqrt{\mu}$. At the same time the electron energy in the maximum will be of the order of mU_0^2, where m is the ion mass, and U_0 — the Alfvén wave velocity. Meanwhile, the perturbed field H will be of the order of $\sqrt{\mu}H_0 \ll H_0$. Therefore, quite weak field oscillations are capable of accelerating electrons to high energies.

For sufficiently great dimensions of the system, powerful short pulses may form out of initially weak pulses of long extent, which assure a still more intensive electron acceleration. An instability may appear on account of the relative motion of electrons and ions [3]. If the instability develops during the time , much lesser than the time of the pulse's passing its own length, the presence of the above-examined waves must lead to a strong heating of the plasma's electron component, without heating of the ionic component.

Such phenomena offer interest for the theory of the origin of the Earth's outer radiation belt, within which only fast electrons could have been detected to-date [4].

In conclusion, I express my deep gratitude to Academician M. A. Leontovich for the discussion of the present work.

*** E N D ***

Institute of Nuclear Physics
Moscow State University

Entered on 25 Sept.1961

REFERENCES

1. R. Z. Sagdeyev. Sb."Fizika plazmy i problema upravlyayemykh termoyadernykh reaktsiy. Izd. A.N.SSSR, p.358, 1958.

2. D. Gardner, G. Gertsel', G. Grad, S. Moravets, M. Rouz, G. Rubin. Tr.II Zhenevskoy konferentsii po mirnomu ispol'zovaniyu atomnoy energii. Doklady inostrannykh uchenykh. Fizika goryachey plazmy i termoyadernyye reaktsii. GLAVATOM, p. 94, 1959.

3. A. A. Vedenov, E. P. Velikov, R. Z. Sagdeyev. UFN, 72, 653, 1961

4. N. S. Vernov, A. E. Chudakov. UFN, 70, 585, 1960.

Translated by ANDRE L. BRICHANT
for the
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

13 May 1962.